Readiness in Formation Control of Multi-Robot System

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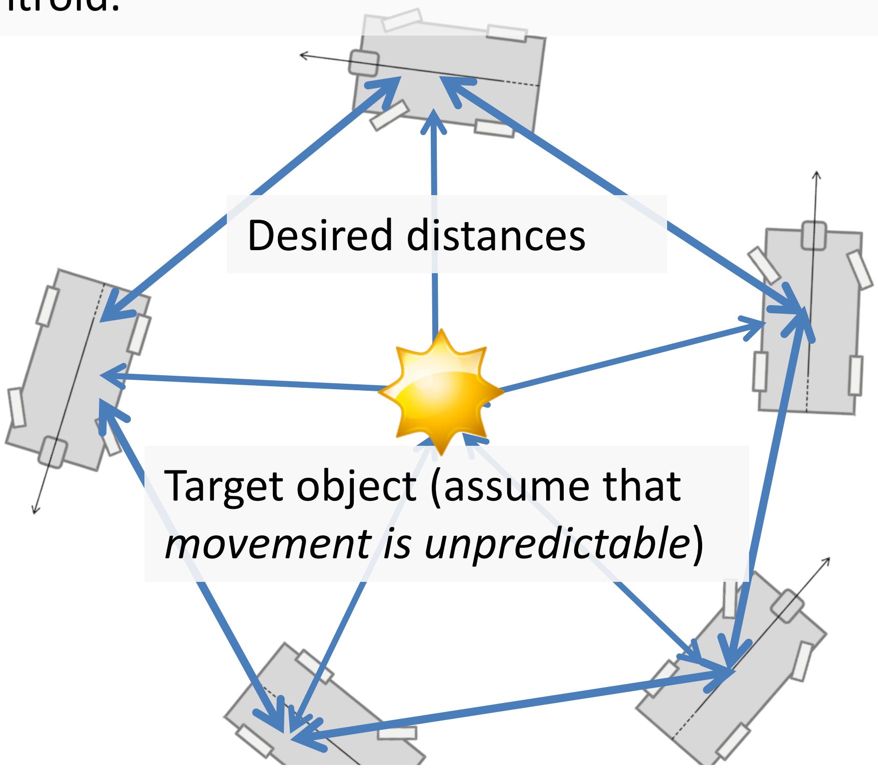
Motivation

<u>Scenario:</u>
Vehicles maintain an equally distributed formation on a circle with a moving object always at its centroid.



What motion of the vehicles is the best to track the target object in formation?

Spiral motion?

Which initial headings give the best response to the arbitrary movement of the target object?

How much the formation is “ready” for any perturbation?

Outline

- Formation control
  - Distance-based formation control
  - Unicycle model for individual agents
- Readiness
  - Definition from a general viewpoint
  - Readiness optimization
- Case study
  - Formation control with unicycle models
  - Optimal readiness
- Conclusion

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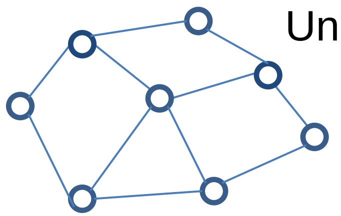
Distance-Based Formation Control

- Interaction rule for follower agent *i* :



$p_i \in \mathbb{R}^2$: position

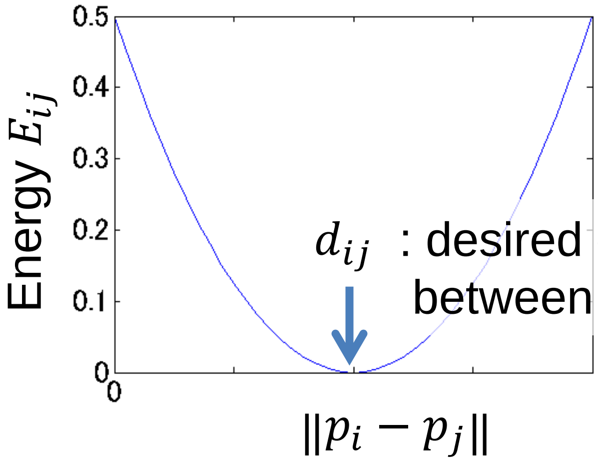
$$\dot{p}_i(t) = -\sum_{j\in\mathcal{N}(i)} \frac{\partial E_{ij}(\|p_i - p_j\|)^T}{\partial p_i} = \underbrace{-\sum_{j\in\mathcal{N}(i)} w_{ij}(\|p_i - p_j\|)(p_i - p_j)}$$

E_{ij}: pair-wise *edge-tension energy*

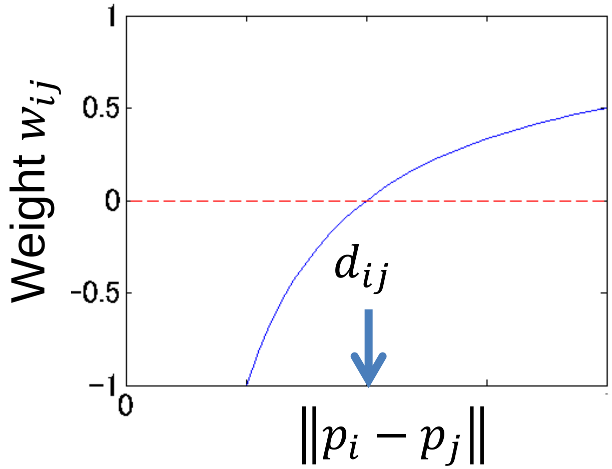
$E_{ij}(\|p_i - p_j\|) = 0$ if $\|p_i - p_j\| = d_{ij}$

weighted consensus protocol

(w_{ij} depends on $\|p_i - p_j\|$)



d_{ij} : desired distance between *i* and *j*



Leader-Follower Formation Control

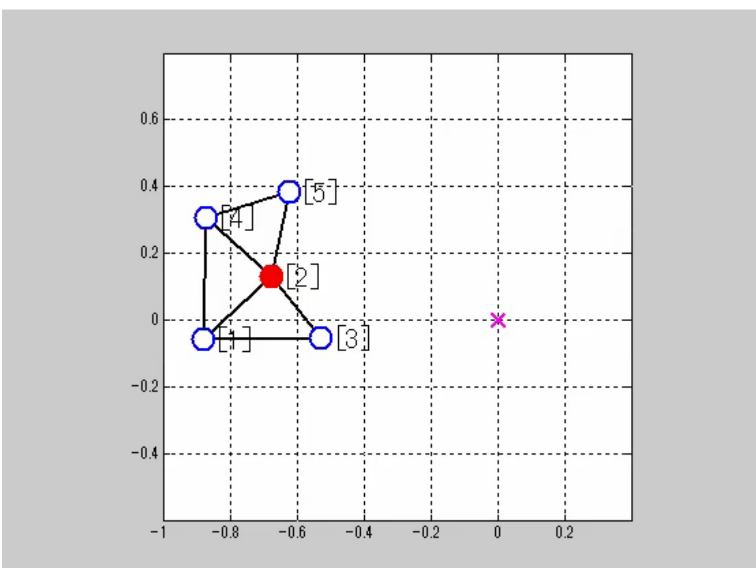
- Assume one agent (leader) can be arbitrarily controlled
- Remaining agents (followers) obey the original interaction rule

$$p = \begin{bmatrix} \boxed{p_1} \\ \boxed{p_2} \\ \vdots \\ \boxed{p_N} \end{bmatrix} \qquad \boxed{p_f} \in \mathbb{R}^{2N_f} \ (N_f \text{ followers}) \qquad \boxed{p_\ell} \in \mathbb{R}^2$$

Single integrator

$$\dot{p}_f(t) = -\frac{\partial E(p_f, p_\ell)}{\partial p_f}^T$$

$\dot{p}_\ell(t)$: input to the network



$$E(p(t)) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} E_{ij}(\|p_i - p_j\|)$$

$E_{ij} = 0$ if $(\mathrm{i}, \mathrm{j}) \notin \mathbf{E}$ (edgeset)

Unicycle Model

- Model

$$\dot{p}_i = \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix} v_i$$

$$\dot{\theta}_i = \omega_i$$

i: follower's ID ($i = 1, \dots, N_f$)

p_i : position
θ_i : heading direction

v_i : linear velocity
ω_i : angular velocity

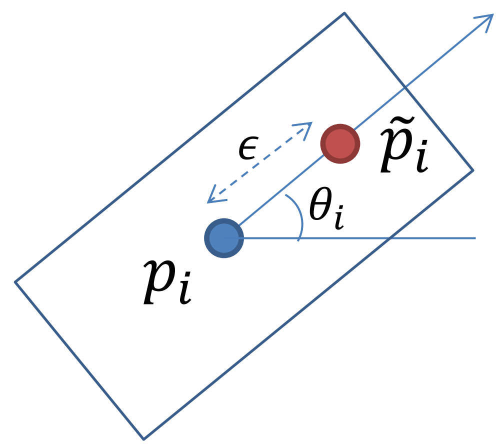
We want to realize

$$\dot{p}_i = \boxed{\mu_i \triangleq -\frac{\partial E}{\partial p_i}^\top (p)}$$

- Off-center point control

$$\tilde{p}_i = p_i + \epsilon \begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}$$

Consider $\dot{\tilde{p}}_i = \mu_i$
instead of $\dot{p}_i = \mu_i$



$$\dot{\tilde{p}}_i = \dot{p}_i + \epsilon \begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \end{bmatrix} \dot{\theta}_i$$

$$= \begin{bmatrix} \cos\theta_i & -\sin\theta_i \\ \sin\theta_i & \cos\theta_i \end{bmatrix} \begin{bmatrix} v_i \\ \epsilon\omega_i \end{bmatrix}$$

$$\therefore \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} = \begin{bmatrix} \cos\theta_i & \sin\theta_i \\ -\frac{1}{\epsilon}\sin\theta_i & \frac{1}{\epsilon}\cos\theta_i \end{bmatrix} \boxed{\mu_i}$$

Dynamics of Unicycle Formation Control

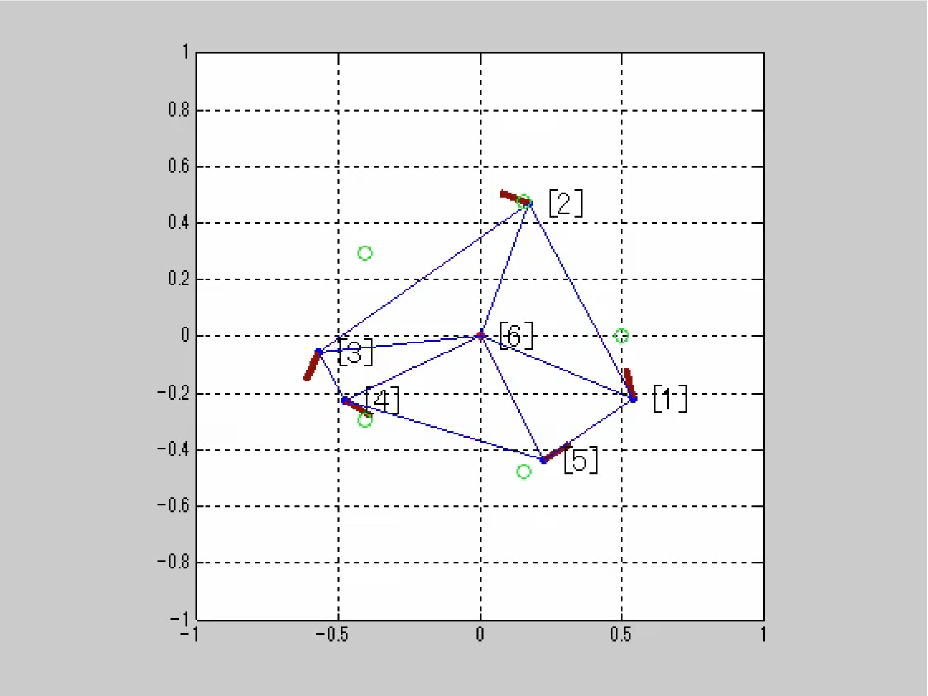
- Dynamics (stacked vector form)

$$\begin{cases} \dot{p}_f = f_p(p_f, \theta_f, p_\ell) \triangleq -S_1(\theta_f) S_1(\theta_f)^\top \dfrac{\partial E}{\partial p_f}^\top (p_f, p_\ell) \\ \dot{\theta}_f = f_\theta(p_f, \theta_f, p_\ell) \triangleq -\dfrac{1}{\epsilon} S_2(\theta_f)^\top \dfrac{\partial E}{\partial p_f}^\top (p_f, p_\ell) \end{cases}$$

$$\theta_f = [\theta_1, \ldots, \theta_{N_f}]^\top$$

$$S_1(\theta_f) \triangleq \operatorname{blockdiag}\left(\begin{bmatrix} \cos\theta_i \\ \sin\theta_i \end{bmatrix}\right)$$

$$S_2(\theta_f) \triangleq \operatorname{blockdiag}\left(\begin{bmatrix} -\sin\theta_i \\ \cos\theta_i \end{bmatrix}\right)$$



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Properties for Leader-Follower Formation Control

- Point-to-point (formation control) property
  - Controllability [Rahmani,Mesbahi&Egerstedt SIAM09]

Characterized by network topologies

- Instantaneous/short-term response of formation
  - Manipulability [Kawashima&Egerstedt CDC11]
  - Responsiveness [Kawashima,Egerstedt,Zhu&Hu CDC12]

Characterized by network topologies and agent positions

☹ Dependent on the single-integrator model of mobile agents.

How to characterize the headings of nonholonomic (e.g. unicycles) agents in terms of shot-term response?

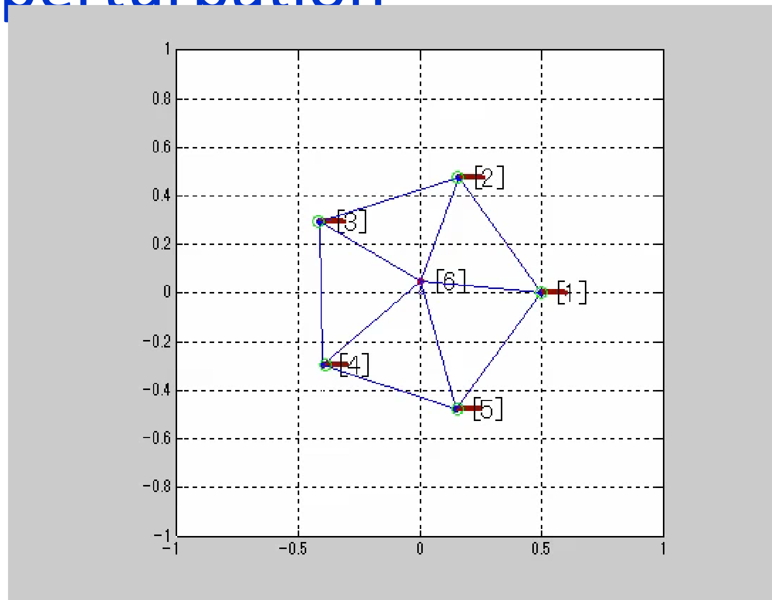
Readiness of multi-robot formation

Response to the leader's perturbation

- Dynamics (stacked vector form)

$$\begin{cases} \dot p_f = f_p(p_f,\theta_f,p_\ell) \triangleq -S_1(\theta_f)S_1(\theta_f)^\top \dfrac{\partial E}{\partial p_f}^\top (p_f,p_\ell) \\ \dot\theta_f = f_\theta(p_f,\theta_f,p_\ell) \triangleq -\dfrac{1}{\epsilon} S_2(\theta_f)^\top \dfrac{\partial E}{\partial p_f}^\top (p_f,p_\ell) \end{cases}$$

$$\dot x = f(x(t),u)$$



- Initial condition $p^* = \begin{bmatrix} p_f^* \\ p_\ell^* \end{bmatrix}$ satisfies desired distances

$$\begin{bmatrix} p_f(0) \\ p_\ell(0) \end{bmatrix} = \begin{bmatrix} p_f^* \\ p_\ell^* + \boxed{\delta p_\ell} \end{bmatrix} \quad \delta p_\ell = \gamma \begin{bmatrix} \cos\theta_\ell \\ \sin\theta_\ell \end{bmatrix}$$

Perturbation on the leader (fixed over $t \in [0,T]$)

$$\theta_f(0) = \boxed{\theta_{f0}}$$ Initial headings of the followers

$$x(0) = x_0$$

We want to find best θ_{f0} that minimizes the edge-tension energy $E(p(T))$ for arbitrary θ_ℓ

$$x_0^\star = \mathrm{argmin}_{x_0} J(x_0)$$

$J(x_0)$: cost functional

Readiness from general viewpoint

"*Readiness*" is an index to describe how well the initial condition is prepared for a variety of possible disturbances (inputs)

(Ex.) Unicycle formation case

$$x_0 \triangleq \theta_f(0)$$

$$u \triangleq \theta_\ell, U \triangleq [0, 2\pi]$$

- Given the dynamics of the agents:

$$\dot{x} = f(x(t), u)$$

with the initial condition $x(0) = x_0 \in \mathbb{R}^d$,
where $u \in U$ is an exogenous input which is constant in short interval $[0, T]$

- Readiness for the response in interval $[0, T]$ is characterized by

$$J(x_0) = \int_U \left(\int_0^T L(x(t), u)dt + \Psi(x(T), u) \right) du.$$

(Ex.) Unicycle formation case $L(x, u) = 0, \quad \Psi(x(T), u) \triangleq E(p(T), \theta_\ell)$

Readiness Optimization

- Optimal initial condition x_0^*

$$x_0^* = \arg\min_{x_0} J(x_0) \qquad J(x_0) = \int_U \left(\int_0^T L(x(t), u)dt + \Psi(x(T), u) \right) du.$$

$$\text{subject to} \quad \dot{x}(t) = f(x(t), u)$$
$$x(0) = x_0$$

- Optimality Conditions (derived by the calculus of variations)

$$\boxed{\frac{\partial J}{\partial x_0} = \int_U \lambda^\top(0, u) du = 0}$$

Optimization via gradient descent:

$$x_0^{(k+1)} = x_0^{(k)} - \eta \frac{\partial J}{\partial x_0}^\top$$

Costate equation

$$\begin{cases} \dot{\lambda}(t, u) &= -\dfrac{\partial L}{\partial x}^\top (x(t), u) - \dfrac{\partial f}{\partial x}^\top (x(t), u) \lambda(t, u) \\ \lambda(T, u) &= \dfrac{\partial \Psi}{\partial x(T)}^\top (x(T), u) \end{cases}$$

with the costate $\lambda \in \mathbb{R}^{dN}$.

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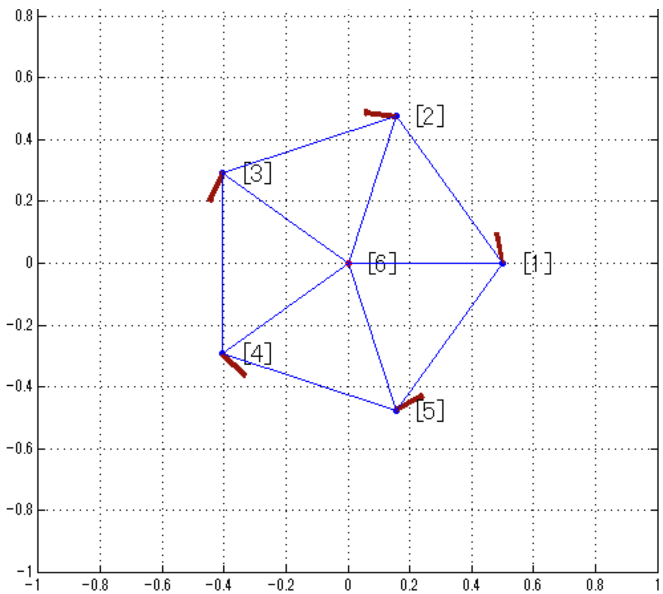
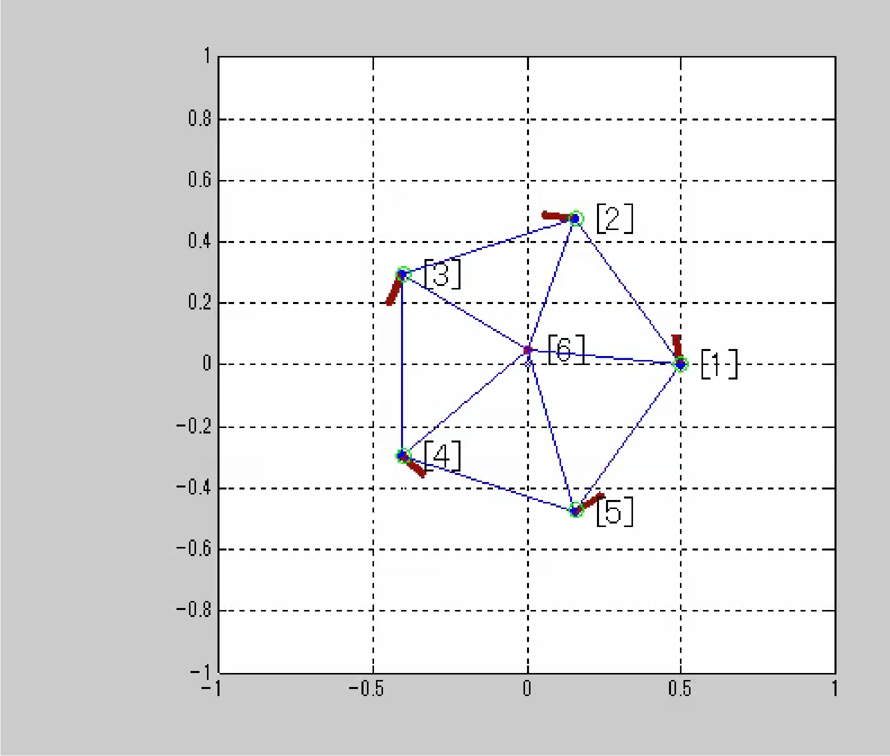
Case Study: Unicycle formation

- Perturbation (input) on leader $\delta p_\ell = \gamma \begin{bmatrix} \cos\theta_\ell \\ \sin\theta_\ell \end{bmatrix}$, $\theta_\ell \in [0, 2\pi]$
- Optimization of followers headings

$$\min_{\theta_{f0}} J(\theta_{f0}) = \int_0^{2\pi} \underbrace{E(p(T), \theta_\ell)}_{\text{Terminal cost}} d\theta_\ell$$

Gradient descent started by tangential θ_f

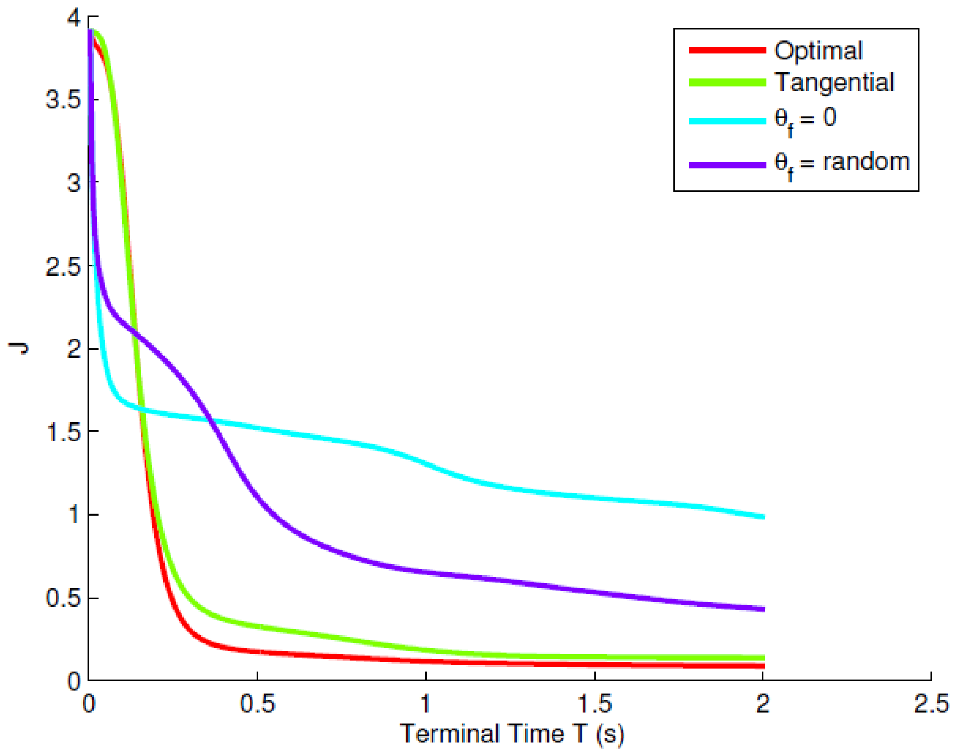




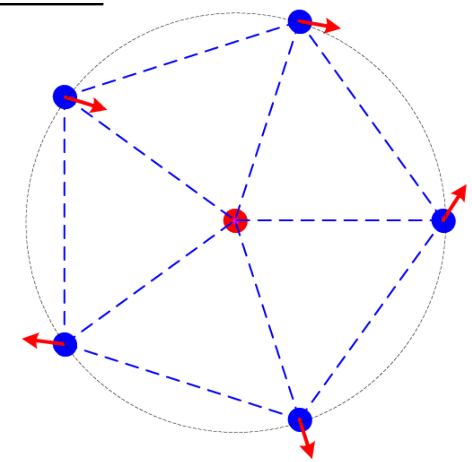
Cost Comparison (1): Cost *J* for Readiness

- Comparison of cost *J* with other headings

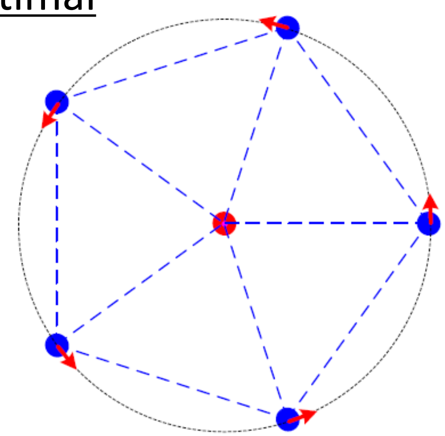
$$J(\theta_{f0}) = \int_0^{2\pi} E(p(T), \theta_\ell) d\theta_\ell$$



Random

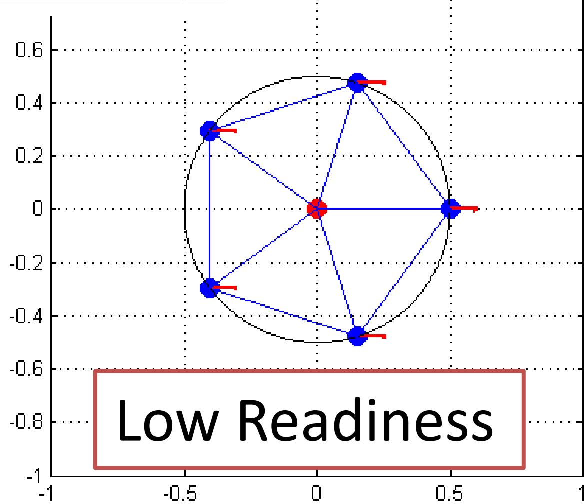


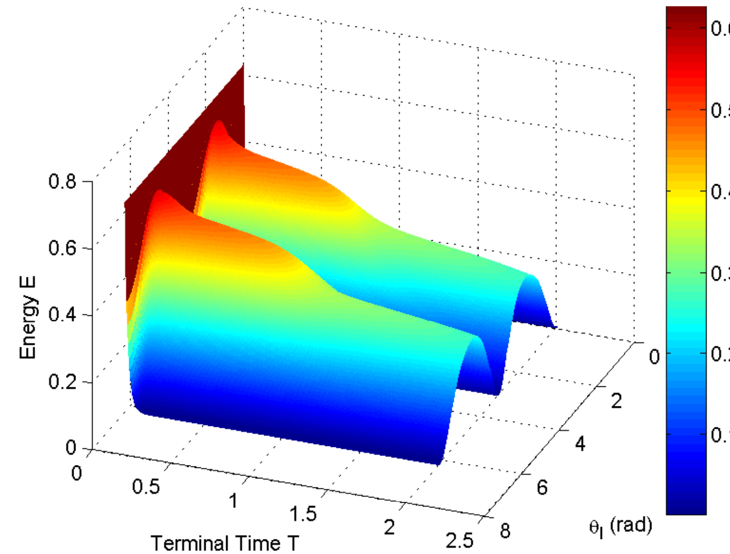
Optimal



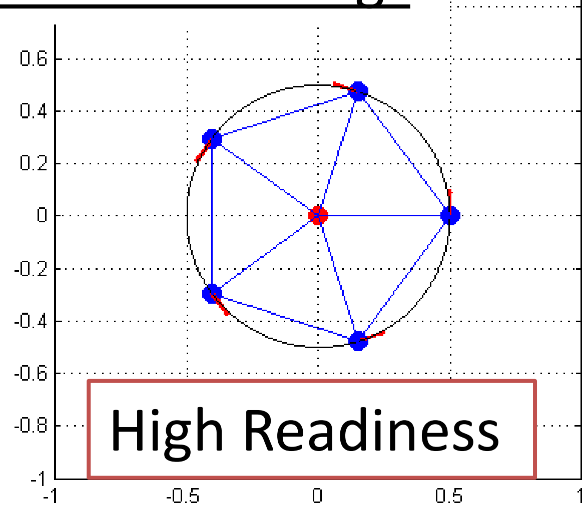
Cost Comparison (2): Distribution over θ_ℓ

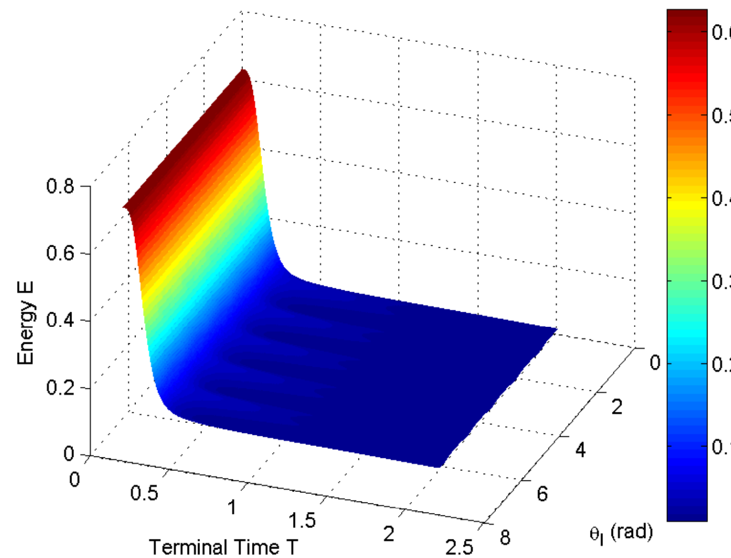
Uniform Headings





Optimal Initial Headings





Conclusion

- *Readiness* notion to characterize the initial condition of nonholonomic multi-agent systems
- Optimality condition (first-order necessary condition)

❑ Future Work

- Utilize the readiness to optimize both headings and positions
- Investigate optimization algorithm which can find global minima

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